Math

A Study of Romance: Using a System of Differential Equations

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Love is by no means an easy thing to measure. By using a system of differential equations and set parameters, we will observe the changes in behavior of two lovers' feelings for each other. We will show graphs of these events, and describe what kind of behavior each graph has about the origin. Using this information, we will be able to determine what outcome the relationship will most likely have.

Introduction

In Strogatz' article (3), he proposes the idea of using differential equations to predict the outcome of a relationship. Sprott's article (2), takes this idea and he dives into a field that is rarely explored: using mathematics to describe and predict the outcomes of a romantic relationship. He does this by using differential equations and changing parameters depending on how the two feel for one another.

The concept of love is difficult to measure because it is a complex matter. In the case for this project, we will explore only the feelings of two individuals that are in a relationship. All outside people, such as: family, friends, and other love interest, will be completely omitted from this study. This would cause new parameters to occur, and in turn, a totally different set of differential equations to form. For now, only the two people whose feelings are being discussed will exist.

Consider the relationship between two people. There is never a certain outcome that will occur from this love affair. The way the two feel for one another will, in most circumstances, change over time. Whether they fall deeply in love, begin to hate each other, or even if their feelings just go away with time; there is no way to ever be completely certain of the outcome.

There is a way to calculate the likeliness of how the two's relationship will turn out. We can use a system of differential equations to help determine if the couple will forever be in love or if they will eventually have a huge fall out.

Process

In order to solve these problems, the following set of differential equations will be used to solve all five of the examples that will be discussed in this paper.

$$\frac{dX}{dt} = aX + bY$$

$$\frac{dY}{dt} = cX + dY$$

In the previous system of differential equations,

- •X represents the effect of the male's feelings.
- •Y represents the effect of the female's feelings.
- •The parameter a represents how the male is encouraged by his own feelings.
- •The parameter b represents how his feelings are affected by the female's feelings (2).
- •The parameter c represents how the female's feelings are affected by the male's feelings.
- •The parameter d represents how the female is encouraged by her own feelings (2).

In order to work with the parameters, they were converted into this:

$$\frac{d}{dt}\begin{bmatrix} X \\ Y \end{bmatrix} = A\begin{bmatrix} X \\ Y \end{bmatrix}$$

where:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The initial conditions for all of the examples will be: X0 = Y0 = 1. This is because we are only concerned with what is occurring in the first quadrant. If X(t) and Y (t) --> $-\infty$ as t --> ∞ then the relationship will become hatred. Also if X(t) and Y (t) --> 0 as t --> ∞ then the relationship will neutral out.

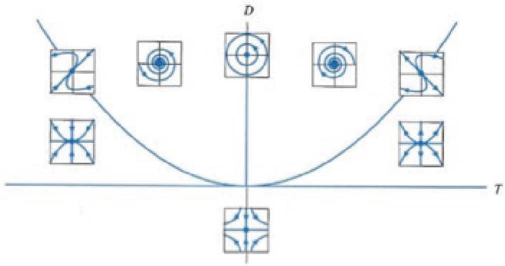

Figure 1: The trace determinant plane [1].

Trace Determinant Plane

Instead of finding the eigenvalues, the trace determinant plane can be used to summarize the different behavior for linear systems in just one picture. The quantity of T = a + d is called the trace of a matrix, and the quantity of D = ad – bc is the determinant of the matrix. These are then placed into Equation (1) to find the eigenvalues. However, we can determine the behavior of a linear system without computing the eigenvalues if we know the T and D.

$$\lambda = \frac{-T \pm \sqrt{T^2 - 4D}}{2} \quad (1)$$

Then the only thing that we are interested in is the discriminant. Setting the discriminant equal to 0, will provide the solution D = $\frac{T^2}{4}$, which is the parabola that can be see in Figure 1. There are three different scenarios that come from the trace-determinant plane:

- D = $\frac{T^2}{4}$ which implies that the solution lines on the line, and have one real solution.
- D > $\frac{T^2}{4}$ which implies that the solution will have two complex solu tions.
- D < $\frac{T^2}{4}$ which implies that the solution will have two real solutions.

Depending on where (T,D) lies on the trace-determinant plane, we can determine the behavior of the graph.This graph was used to determine the behavior of the examples in this paper.

Using all of the previously stated information, the following examples will be solved. By converting the parameters into this matrix, the behavior of the eigenvalues will be seen. The trace-determinant plane can then be used to determine the behavior of the differential equations about the origin, and also the phase portrait of the system can be created. The collection of this information will be used to determine the potential outcome of the couple's relationship.

Examples

This section will consist of the five couples that were chosen to be examined, the set parameters that were determined and why, the phase portrait given by the parameters, the behavior of the graph as t ∞, and finally, the outcome of their relationship that can be concluded using the parameters and phase portrait.

Romeo and Juliet

The first sample we will examine is the famous couple Romeo and Juliet. Their relationship ended up coming to a tragic end, but their love for each other knew no bounds. While taking this into consideration, parameters will be determined which will reflect their endless love for each other.

Parameters

These parameters were chosen to represent Romeo and Juliet's feeling towards each other:

$$A = \begin{bmatrix} 1 & 0.5 \\ 0.2 & 2 \end{bmatrix}$$

with the parameters meaning:
a = 1, represents Romeo's initial love for Juliet.
b = 0.5, represents how Juliet's love for Romeo positively affects his love for her.
c = 0.2, represents how Romeo's love for Juliet has a positive effect on her love for him.
d = 2, represents Juliet's initial love for Romeo.

These values for the parameters will show how Romeo and Juliet's love could have potentially been, if they had not met their end at such an early age. Next, we will see what the phase portrait looks like and analyze the behavior of it.

Behavior and Outcome
The values T = 3 and D = 1.9, imply that D < . The parameters of Romeo and Juliet's romance create a source about the origin, which can be seen in Figure 2. By examining the first quadrant, X(t) and Y (t) --> ∞ as t --> ∞.

This is to say that their relationship will last; in fact, their love for each other will increase rapidly for each other over time.

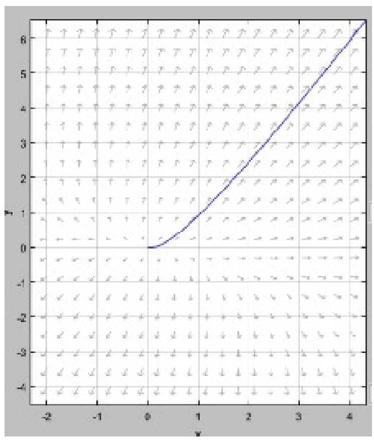

Figure 2: The phase portrait of Romeo and Juliet's set parameters.

Ross and Rachel

Ross and Rachel are the famous couple from the television show Friends; they are known for their "on and off" relationship. Ross is always interested in Rachel, but the same cannot always be said for her. Initially Rachel does not view Ross in a romantic light, but as time goes on she begins to see him differently.

Parameters
Due to this, the following parameters were chosen to represent their relationship:

$$A = \begin{bmatrix} 2.5 & 2 \\ 1.2 & -0.2 \end{bmatrix}$$

a = 2.5, represents Ross' initial love for Rachel.
b = 2, represents how Ross reacts positively for Rachel's love for him.
c = 1.2, represents how Ross' love for Rachel has a positive effect on her love for him.
d = -0.2, represents Rachel's disinterest in Ross initially.

These values will demonstrate the "on and off" relationship that they shared. By looking at the phase portrait in Figure 3, we can see the behavior that this causes.

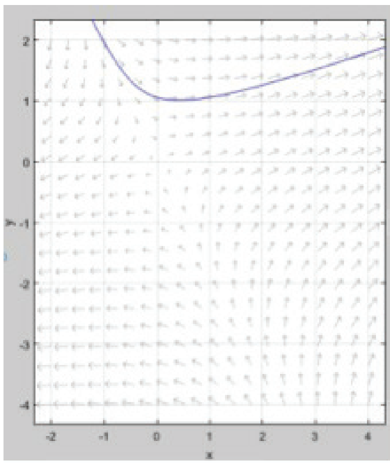

Figure 3: The phase portrait of Ross and Rachel's set parameters.

Behavior and Outcome

The values T = 2.3 and D = –2.4, imply that D $\frac{T^2}{4}$ < . .As t --> ∞, X(t) and Y (t) will also --> ∞ This can be seen in Figure 3, and it also shows how these parameters create a saddle about the origin.

However, we are only interested in the outcome within the first quadrant, meaning that their relationship will last. It will never neutral out or tend towards –∞.

Harry and Cho

In the film *Harry Potter*, Harry has a crush on Cho, but she is not interested in him at first. Her feelings for him begin to increase, but his feeling for her start to dissipate. These were the credentials that the parameters were chosen on to depict this relationship.

Parameters

$$A = \begin{bmatrix} 0.5 & -1 \\ 2 & -0.6 \end{bmatrix}$$

a = 0.5, shows Harry's initial interest in Cho.
b = -1, represents how Harry's love for Cho begins to dissipate as her interest in him increases.
c = 2, shows how Harry's feelings for Cho have a positive effect on his feelings towards him.
d = -0.6, represents Cho's initial disinterest in Harry.
These parameters will demonstrate how Harry begins to lose his feelings for Cho as she begins to like him more. Similarly, it also shows how Cho begins to become interested in Harry as he becomes disinterested in her.

Behavior and Outcome

The values for T and D are: T = –0.1 and D = 1.7, this implies that D > $\frac{T^2}{4}$ . .As t --> ∞, X(t) and Y (t) --> 0. This can be seen in Figure 4, and it is also show that these parameters create a spiral sink about the origin.

However, we are only interested in the outcome within the first quadrant, which implies that their relationship does not tend towards –∞. Therefore, it does not become hate but instead becomes neutral. Harry and Cho will neither hate nor love each other, and their relationship will not last.

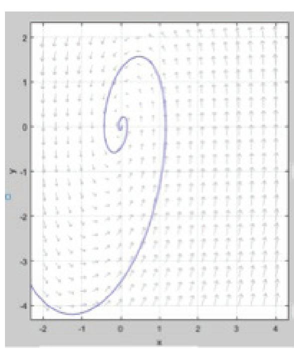

Figure 4: The phase portrait of Harry and Cho's set parameters.

Han Solo and Leia

From the hit movie saga Star Wars, this couple's feelings oscillate between love and hate quite often. This was a more difficult couple to try to pick parameters for, but I ended up using the fact that Han Solo tends to not be interested in Leia as much as she is in him. This, as we know, is just a facade he puts off because he has that "bad boy" aura about him; the same can be said for visa versa. However, they do have loving feelings toward each other some of the time. Using all of this I chose the following parameters.

Parameters

$$A = \begin{bmatrix} -0.5 & -2 \\ 0.8 & 0.8 \end{bmatrix}$$

a = -0.5, represents the disinterest that Han Solo originally has for Leia.
b = -2, represents how Han Solo's love for Leia is affected by Leia's interest in him.
c = 0.8, shows how Han Solo's feeling for Leia positively affect her feelings towards him.
d = 0.8, represents Leia's initial feelings for him.

These values show how Han Solo acts as if he is not interested in Leia, and this positively affects her feelings towards him. It also shows that as Leia likes Han Solo more and more, he acts even more disinterested in her. This behavior has an interesting behavior which can be seen in Figure 5.

Behavior and Outcome

The values for T and D are: T = 0.3 and D = 1.2, this implies that D > $\frac{T^2}{4}$. These parameters create a spiral source about the origin, which can be seen in Figure 5, and actually creates an oscillation going back and forth between like and dislike or love and hate. This implies that as t --> ∞, X(t) and Y (t) oscillate continuously.

They will never feel neutral towards each other, and their feelings will be mutually loving for about one fourth of the time. This implies that their relationship will, in fact, last.

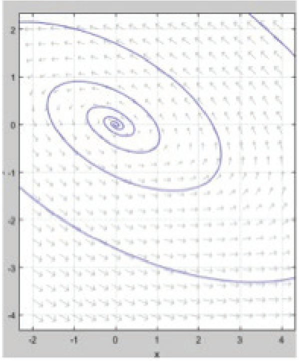

Figure 5: The phase portrait of Han Solo and Leia's set parameters.

Ron Swanson and Tammy

Descending from the popular television show *Parks and Recreation*, this couple is divorced and sometimes find themselves having a little fling. However, whenever they do get together the outcome is never good. Neither Ron or Tammy like each other at all. In fact, they usually hate each other. However, they both respond positively to the negative emotions towards each other, and this turns into some dramatic chaos occurring.

Parameters

$$A = \begin{bmatrix} -2 & 2 \\ 1.7 & -2 \end{bmatrix}$$

a = -2, shows Ron's initial hate for his ex-wife Tammy.
b = 2, represents how Ron's feelings for Tammy make him fall for her again.
c = 1.7, shows how Tammy's feelings for Ron make her also fall for him.
d = -2, represents Tammy's initial dislike for Ron.
These parameters demonstrate that Ron initially hates Tammy, and she also similarly hates him to begin with. As the other dislikes each other more, this has a positive feeling on the other person. This chaotic behavior can be seen in Figure 6.

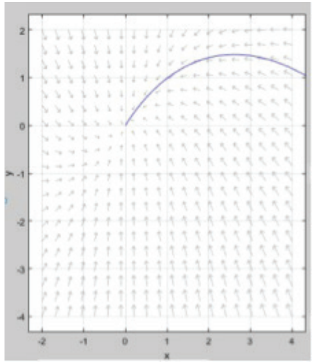

Figure 6: The phase portrait of Ron and Tammy's set parameters.

Behavior and Outcome

The values for T and D: T = –4 and D = 0.6. This implies that $D < \frac{T^2}{4}$, which can then be used to show that these parameters create a sink about the origin. As t --> ∞, X(t) and Y (t) --> –∞. Their relationship will actually last, however, they will both have very strong feelings of hate for each other.

Conclusion

All of the hypothetical situations presented were based upon famous Hollywood couples, and the outcome of their relationships that were predicted are accurate to what actually occurred.

This process is of course only hypothetical because romance is actually a rather complex thing. A relationship is not only dependent upon the two people involved, but family and friends can sometimes have a big impact on the future of the couple. Also, an individual's feelings for his or her lover is rarely a consistent thing, but can change depending on what is happening in one another's lives. There are many different variables that ultimately have an impact on a relationship's future. We used only four set parameters in this project, and assumed that they would not vary.

Even though these examples are simplifications, they still assume that love is a simple scalar variable where the individuals involved respond in a consistent way to their own love and to the others toward them without external reference [2]. Through the use of this information we can conclude that by giving a set of parameters, a prediction of the potential outcome of a relationship can be determined.

References

[1] Blanchard, Paul., Devaney, Robert, L., and Hall, Glen R., "Trace Determinant Plane." Differential Equations. 4th ed. Boston: Cengage Learning. (2012): 347-357.

[2] Sprott, J. C. Dynamical models of love. Nonlinear dynamics, psychology, and life sciences 8.3 (2004): 303-314.

[3] Strogatz, Steven H. Love affairs and differential equations. Mathematics Magazine 61.1 (1988):35.